

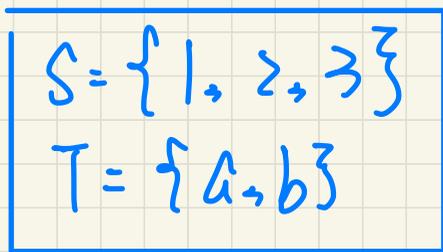
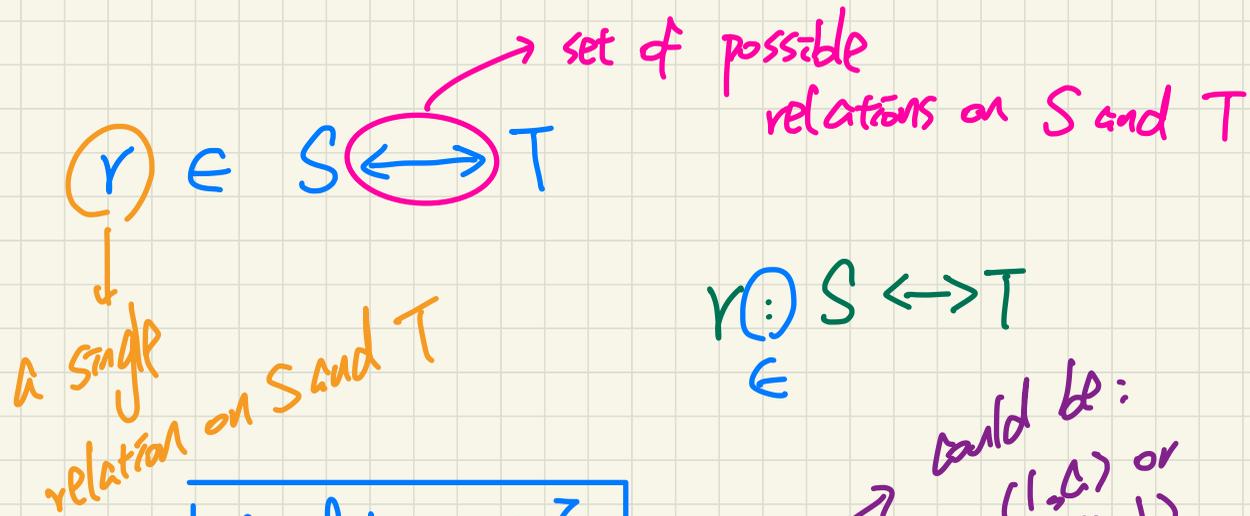
Lecture 6 - January 26

Math Review

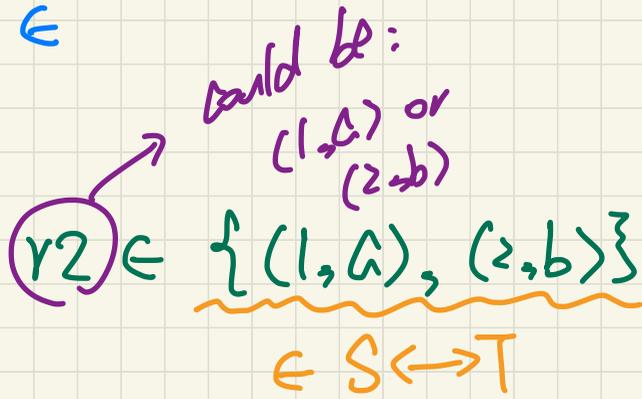
Relations, Relational Operations

Announcement

- **Lab1** submission due in a week
 - + Help: scheduled office hours & TA
 - + tutorial videos
 - + problems to solve
 - + Study along with the Math Review lecture notes.



$\gamma \in S \leftrightarrow T$



$\gamma \in S \leftrightarrow T$

- γ could be:
- (1) \emptyset
 - (2) $S \times T$
 - (3) $\{(1, a), (2, b)\}$

Set of Possible Relations

- **Set** of possible relations on S and T:
- Dedicated symbol for **set** of possible relations on S and T:
- Declare that set r is a relation on S and T:

Example: Enumerate all relations on {a, b} and {2, 4}.

Hint: How many?

rad: $2^4 = 16$

$\{a, b\} \leftrightarrow \{2, 4\}$

$$\mathcal{P}(\{a, b\} \times \{2, 4\})$$

max relation on the two sets

$$\mathcal{P}(\{(a, 2), (a, 4), (b, 2), (b, 4)\})$$

$$= \left\{ \begin{array}{l} \emptyset, \text{ 1* relation of size } 0 \times 1 \\ \{(a, 2)\}, \{(a, 4)\}, \{(b, 2)\}, \{(b, 4)\} \\ \{(a, 2), (a, 4)\}, \{(a, 2), (b, 2)\}, \{(a, 2), (b, 4)\}, \{(a, 4), (b, 2)\}, \{(a, 4), (b, 4)\}, \{(b, 2), (b, 4)\} \\ \{(a, 2), (a, 4), (b, 2), (b, 4)\} \end{array} \right.$$

$$\binom{4}{1} = 4$$

1* rel of size 1*1

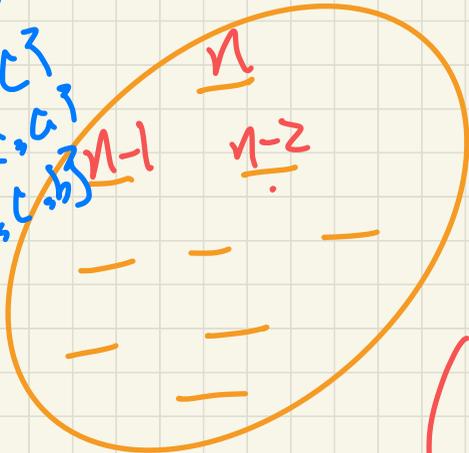
(a) relations of size 2 $\binom{4}{2} = \frac{4 \times 3}{2!} = 6$
 (b) relations of size 3 $\binom{4}{3} = \frac{4 \times 3 \times 2}{3!} = 4$

1* relation of size 4*1

$$\binom{n}{r}$$

out of n given elements,
 how many ways to make
 a set of card. r

$\begin{matrix} a \\ b \\ c \end{matrix}$
 $\begin{matrix} a \\ b \\ c \end{matrix}$



set of
 card. r

$$n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot (n-r+1)$$

$$r!$$

disregard
 duplicates

$$\binom{n}{r} = \frac{n!}{(n-r)! r!}$$

Departure = { toronto, montreal, vancouver }

Destination = { beijing, seoul, penang }

airline \in Departure \leftrightarrow Destination

\hookrightarrow task: enumerate!

Relational Operations: Domain, Range, Inverse

$$r = \{(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)\}$$

$$\text{dom}(r) = \{a, b, c, d, e, f\}$$

$$r = \{(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)\}$$

$$\text{ran}(r) = \{1, 2, 3, 4, 5, 6\}$$

$$r = \{(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)\}$$

$$r^{-1} = \{(1, a), (2, b), (3, c), (4, a), (5, b), (6, c), (1, d), (2, e), (3, f)\}$$

$$|r| = |r^{-1}|$$

→ algebraic properties

Exercise: Relate the domains and ranges of r and its inverse.

$$(1) \text{ dom}(r) = \text{ran}(r^{-1}) \quad (2) \text{ ran}(r) = \text{dom}(r^{-1})$$

Relational Operations: Image

$$* r \in S \leftrightarrow T$$
$$r[s] \text{ assumption: } s \subseteq S$$

$$r = \{(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)\}$$

$v \in \text{Alphabet}$
all lower & upper letters

$r[S]$
a set!

$$r[\{a, b\}] = \{v \mid (d, v) \in r \wedge d \in \{a, b\}\}$$
$$= \{1, 2, 4, 5\}$$

Exercises

- Image of $\{a, b\}$ on r ?
- Image of $\{1, 2\}$ on r ? *undefined
- Image of $\{1, 2\}$ on the inverse of r ?
- Calculate r 's range via an image.
- Calculate r 's domain via an image.

e.g. $r[\emptyset] = \emptyset$
e.g. $r[\{x, y\}] = \emptyset$
 $\rightarrow \{a, b, d, e\}$

$$\text{ran}(r) = r[\text{dom}(r)]$$

$r[\{1, a\}]$ x undefined

$$\text{dom}(r) = \text{ran}(r^{-1})$$

dom(r^{-1})

$$\underline{r} \in S \leftrightarrow T \quad s \subseteq S$$

	domain	range
Restriction	$s \triangleleft r$	$r \triangleright s$
Subtraction	$s \triangleleft r$	$r \triangleright s$

Another relation

$$\underline{ds} \triangleleft r = \{ (d, r') \mid (d, r') \in r \wedge d \in ds \}$$

$\underbrace{\hspace{1.5cm}}_{\text{domain restriction}}$

Relational Operations: Restrictions vs. Subtractions

$$r = \{(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)\}$$

$$\{a, b\} \triangleleft r = \{(a, 1), (b, 2), (a, 4), (b, 5)\}$$

$$r = \{(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)\}$$

$$r \triangleright \{1, 2\} = \{(a, 1), (b, 2), (d, 1), (e, 2)\}$$

$$r = \{\langle a, 1 \rangle, \langle b, 2 \rangle, (c, 3), \langle a, 4 \rangle, \langle b, 5 \rangle, (c, 6), (d, 1), (e, 2), (f, 3)\}$$

$$\{a, b\} \triangleleft r = \{(c, 3), (c, 6), (d, 1), (e, 2), (f, 3)\}$$

$$r = \{\langle a, 1 \rangle, \langle b, 2 \rangle, (c, 3), (a, 4), (b, 5), \langle c, 6 \rangle, \langle d, 1 \rangle, \langle e, 2 \rangle, (f, 3)\}$$

$$r \triangleright \{1, 2\} = \{(c, 3), (a, 4), (b, 5), (c, 6), (f, 3)\}$$

$$V = (s \triangleleft r) \cup (s \triangleleft r)$$

Relational Operations: Overriding

$$r = \{(a, 1), (b, 2), (c, 3), (a, 4), (b, 5), (c, 6), (d, 1), (e, 2), (f, 3)\}$$

Example: Calculate r overridden with $\{(a, 3), (c, 4)\}$

Hint: Decompose results to those in t 's domain and those not in t 's domain.

$$r \triangleleft \underset{\substack{\downarrow \\ \text{a relation}}}{t} = \left\{ (d, v') \mid \underbrace{(d, v') \in t}_{\text{domain}} \vee \underbrace{\left((d, v') \in r \wedge d \notin \text{dom}(t) \right)}_{\text{domain}(t)} \right\}$$

$$\begin{aligned} r \triangleleft \underbrace{\{(a, 3), (c, 4)\}}_t &= \left\{ (d, v') \mid \begin{array}{l} (d, v') \in \{(a, 3), (c, 4)\} \\ \vee \underbrace{(d, v') \in r \wedge d \notin \{a, c\}}_{\text{domain subtraction}} \end{array} \right\} \\ &= \{(a, 3), (c, 4), (b, 2), (b, 5), (d, 1), (e, 2), (f, 3)\} \end{aligned}$$

Problems (don't look at the slides!)

(1) Rewrite the relational image $r[S]$
in terms of dom/ran and/or
restrictions/subtractions.

(2) Rewrite the overriding $r \triangleleft t$
in terms of dom/ran and/or
restrictions/subtractions and/or
set operations.

Lecture 1b

Review on Math: Functions

Functional Property

relation

$$\{ \overset{s}{\underline{a}}, \overset{t1}{1}, \overset{s}{\underline{b}}, \overset{t2}{2}, \overset{s}{\underline{a}}, \overset{t2}{3} \}$$

↳ a relation
not a function!

isFunctional(r) \Leftrightarrow $\in S \leftrightarrow T$

$$\forall \underline{s}, \underline{t1}, \underline{t2} \bullet$$

$$(s \in S \wedge t1 \in T \wedge t2 \in T)$$

\Rightarrow

$$((s, t1) \in r \wedge (s, t2) \in r \Rightarrow t1 = t2)$$

each domain value s maps to at most one range value t

Q: Smallest relation satisfying the functional property.

Q: How to **prove** or **disprove** that a relation r is a function.

Q: Rewrite the functional property using **contrapositive**.